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On Limitations of T Invariance in K Decays

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Abstract

Data from the CPLEAR collaboration coupled with the assumption that the Bell-Steinberger relation holds have provided *direct* evidence for **T** violation. In this note we investigate what we can say about **T** violation *without* such an assumption.

We show that both the modulus and the phase of η_{+-} can be reproduced with **T** *invariant* dynamics through finetuning **CPT** breaking. The large **T** odd correlation observed by the KTeV collaboration in $K_L \rightarrow \pi^+\pi^-e^+e^-$ thus does not yield direct evidence for **T** violation. In such a world the phase of $\frac{\epsilon'}{\epsilon}$ is $\delta_2 - \delta_0 - \phi_{SW} \sim -(85.5 \pm 4)^\circ$. Also, $K^\pm \rightarrow \pi^\pm\pi^0$ could exhibit a **CPT** asymmetry of up to $\text{few} \times 10^{-3}$ without upsetting any known bound.

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1 Introduction

CPT symmetry has an impressive *theoretical* pedigree as an almost inescapable consequence of Lorentz invariant quantum field theories. Observation of a **CP** asymmetry is therefore usually seen as tantamount to the discovery of a violation of time reversal invariance **T**. However the *experimental* verification of **CPT** invariance is much less impressive. Furthermore the emergence of superstring theories has opened – by their fundamentally non-local structure – a *theoretical* backdoor through which **CPT** breaking might slip in. This asks for carefully analysing the empirical basis of **CPT** invariance and the degree to which an observable can establish **T** violation *directly*, i.e. without invoking the **CPT** theorem[1]. In addressing this issue, we will rely on as few other theoretical principles as possible: since we view the observation of **CPT** violation as a rather exotic possibility, we believe we should accept other theoretical restrictions very reluctantly only.

Data from the CPLEAR collaboration have provided *direct* evidence for **T** violation[2]. In this note we want to address the following questions:

- To which degree and in which sectors of $\Delta S \neq 0$ dynamics is **T** violated?
- How accurately is the validity of **CPT** invariance established *experimentally*?
- Which conclusions can be drawn *without* invoking the Bell-Steinberger relation.
- Which is the most promising – or the least hopeless – observable for finding **CPT** violations in kaon decays?

The reader might wonder why we are insisting on analyzing **T** symmetry without assuming the Bell-Steinberger relation. After all, it is viewed as just a consequence of unitarity. Yet the following has to be kept in mind: when contemplating the possibility of **CPT** violation – a quite remote and exotic scenario – we should not consider the Bell-Steinberger relation sacrosanct. The latter is based on the assumption that all relevant decay channels are known. Since the major branching

fractions have been measured with at best an error of 1%, some yet undetermined decay mode with a branching fraction of 10^{-3} can easily be hidden [3]. We are *not* arguing that this is a likely scenario – it is certainly not! However we do not view it to be more exotic than **CPT** violation. Then it does not make a lot of sense to us to allow for the latter while forbidding the former.

The paper will be organized as follows: after briefly reviewing the formalism relevant for $K^0 - \bar{K}^0$ oscillations in Sect. 2 we list the direct evidence for **T** being violated in Sect. 3; in Sect. 4 we analyse the phases of η_{+-} and η_{00} ; after evaluating what can be learnt from $K_L \rightarrow \pi^+\pi^-e^+e^-$ in Sect. 5, we give our conclusions in Sect. 6.

2 Formalism

To introduce our notation and make the paper self-contained we shall record here the standard formalism for the neutral K meson system.

2.1 $\Delta S = 2$ Transitions

The time dependence of the state Ψ , which is a linear combination of K^0 and \bar{K}^0 , is given by

$$i\hbar \frac{\partial}{\partial t} \Psi(t) = \mathcal{H} \Psi(t) \quad , \quad \Psi(t) = \begin{pmatrix} K^0(t) \\ \bar{K}^0(t) \end{pmatrix}. \quad (1)$$

The 2×2 matrix \mathcal{H} can be expressed through the identity and the Pauli matrices [4]

$$\mathcal{H} \equiv \mathbf{M} - \frac{i}{2} \mathbf{\Gamma} = E_1 \sigma_1 + E_2 \sigma_2 + E_3 \sigma_3 - iD \mathbf{1}. \quad (2)$$

with

$$\begin{aligned} E_1 &= \text{Re } M_{12} - \frac{i}{2} \text{Re } \Gamma_{12} \quad , \quad E_2 = -\text{Im } M_{12} + \frac{i}{2} \text{Im } \Gamma_{12} \\ E_3 &= \frac{1}{2} (M_{11} - M_{22}) - \frac{i}{4} (\Gamma_{11} - \Gamma_{22}), \quad D = \frac{i}{2} (M_{11} + M_{22}) + \frac{1}{4} (\Gamma_{11} + \Gamma_{22}). \end{aligned} \quad (3)$$

It is often convenient to use instead *complex* numbers E, θ , and ϕ defined by

$$\begin{aligned} E_1 &= E \sin \theta \cos \phi, \quad E_2 = E \sin \theta \sin \phi, \quad E_3 = E \cos \theta \\ E &= \sqrt{E_1^2 + E_2^2 + E_3^2}. \end{aligned} \quad (4)$$

The mass eigenstates are given by

$$\begin{aligned} |K_S\rangle &= p_1 |K^0\rangle + q_1 |\bar{K}^0\rangle \\ |K_L\rangle &= p_2 |K^0\rangle - q_2 |\bar{K}^0\rangle \end{aligned} \quad (5)$$

with the *convention* $\mathbf{CP}|K^0\rangle = |\overline{K}^0\rangle$ and

$$\begin{aligned} p_1 &= N_1 \cos \frac{\theta}{2}, \quad q_1 = N_1 e^{i\phi} \sin \frac{\theta}{2} \\ p_2 &= N_2 \sin \frac{\theta}{2}, \quad q_2 = N_2 e^{i\phi} \cos \frac{\theta}{2} \\ N_1 &= \frac{1}{\sqrt{|\cos \frac{\theta}{2}|^2 + |e^{i\phi} \sin \frac{\theta}{2}|^2}} \\ N_2 &= \frac{1}{\sqrt{|\sin \frac{\theta}{2}|^2 + |e^{i\phi} \cos \frac{\theta}{2}|^2}}. \end{aligned} \quad (6)$$

The discrete symmetries impose the following constraints:

$$\begin{aligned} \mathbf{CPT} \text{ or } \mathbf{CP} \text{ invariance} &\implies \cos \theta = 0, \quad M_{11} = M_{22}, \quad \Gamma_{11} = \Gamma_{22} \\ \mathbf{CP} \text{ or } \mathbf{T} \text{ invariance} &\implies \phi = 0, \quad \text{Im } M_{12} = 0 = \text{Im } \Gamma_{12} \end{aligned} \quad (7)$$

2.2 Nonleptonic Amplitudes

We write for the amplitudes describing decays into final states with isospin I :

$$\begin{aligned} T(K^0 \rightarrow [\pi\pi]_I) &= A_I e^{i\delta_I}, \\ T(\overline{K}^0 \rightarrow [\pi\pi]_I) &= \overline{A}_I e^{i\delta_I} \end{aligned} \quad (8)$$

where the strong phases δ_I have been factored out and find:

$$\begin{aligned} \mathbf{CPT} \text{ invariance} &\implies A_I = \overline{A}_I^* \\ \mathbf{CP} \text{ invariance} &\implies A_I = \overline{A}_I \\ \mathbf{T} \text{ invariance} &\implies A_I = A_I^* \end{aligned} \quad (9)$$

The expressions for η_{+-} and η_{00}

$$\begin{aligned} \eta_{+-} &= \frac{1}{2} \left(\Delta_0 - \frac{1}{\sqrt{2}} \omega e^{i(\delta_2 - \delta_0)} (\Delta_0 - \Delta_2) \right), \\ \eta_{00} &= \frac{1}{2} \left(\Delta_0 + \sqrt{2} \omega e^{i(\delta_2 - \delta_0)} (\Delta_0 - \Delta_2) \right), \\ \Delta_I &= \frac{1}{2} \left(1 - \frac{q_2}{p_2} \frac{\overline{A}_I}{A_I} \right), \quad |\omega| \equiv \left| \frac{A_2}{A_0} \right| \simeq \frac{1}{20}, \end{aligned} \quad (10)$$

are valid *irrespective* of \mathbf{CPT} symmetry.

2.3 Semileptonic Amplitudes

The general amplitudes for semileptonic K decays can be expressed as follows:

$$\begin{aligned}
\langle l^+ \nu \pi^- | \mathcal{H}_W | K^0 \rangle &= F_l(1 - y_l) \\
\langle l^+ \nu \pi^- | \mathcal{H}_W | \bar{K}^0 \rangle &= x_l F_l(1 - y_l) \\
\langle l^- \bar{\nu} \pi^+ | \mathcal{H}_W | K^0 \rangle &= \bar{x}_l^* F_l^*(1 + y_l^*) \\
\langle l^- \bar{\nu} \pi^+ | \mathcal{H}_W | \bar{K}^0 \rangle &= F_l^*(1 + y_l^*).
\end{aligned} \tag{11}$$

with the selection rules

$$\begin{aligned}
\Delta S = \Delta Q \text{ rule:} & \quad x_l = \bar{x}_l = 0 \\
\mathbf{CP} \text{ invariance:} & \quad x_l = \bar{x}_l^*; \quad F_l = F_l^*; \quad y_l = -y_l^* \\
\mathbf{T} \text{ invariance:} & \quad \text{Im } F = \text{Im } y_l = \text{Im } x_l = \text{Im } \bar{x}_l = 0 \\
\mathbf{CPT} \text{ invariance:} & \quad y_l = 0, \quad x_l = \bar{x}_l.
\end{aligned}$$

3 Direct Evidence for \mathbf{T} Violation

The so-called Kabir test[5] represents a quantity that probes \mathbf{T} violation without reference to \mathbf{CPT} symmetry:

$$A_{\mathbf{T}} \equiv \frac{\Gamma(K^0 \rightarrow \bar{K}^0) - \Gamma(\bar{K}^0 \rightarrow K^0)}{\Gamma(K^0 \rightarrow \bar{K}^0) + \Gamma(\bar{K}^0 \rightarrow K^0)} \tag{12}$$

A nonvanishing $A_{\mathbf{T}}$ requires

$$M_{12} - \frac{i}{2}\Gamma_{12} \neq M_{21} - \frac{i}{2}\Gamma_{21}. \tag{13}$$

which constitutes \mathbf{CP} as well as \mathbf{T} violation. Associated production flavor-tags the *initial* kaon. The flavor of the *final* kaon is inferred from semileptonic decays; i.e., we measure the \mathbf{CP} asymmetry

$$A_{\mathbf{CP}} \equiv \frac{\Gamma(K \rightarrow l^- \nu \pi^+) - \Gamma(\bar{K} \rightarrow l^+ \nu \pi^-)}{\Gamma(K \rightarrow l^- \nu \pi^+) + \Gamma(\bar{K} \rightarrow l^+ \nu \pi^-)} \tag{14}$$

Yet a violation of \mathbf{CPT} invariance and/or of the $\Delta S = \Delta Q$ rule can produce an asymmetry in the latter – $A_{\mathbf{CP}} \neq 0$ – without one being present in the former – $A_{\mathbf{T}} = 0$. These issues have to be tackled first. There is nothing new in our remarks on this subject; we add them for clarity and completeness.

Analysing the asymmetries in $\Gamma(\bar{K}^0(t) \rightarrow l^+ \nu K^-)$ vs. $\Gamma(K^0(t) \rightarrow l^- \bar{\nu} K^+)$ and $\Gamma(\bar{K}^0(t) \rightarrow l^- \bar{\nu} K^+)$ vs. $\Gamma(K^0(t) \rightarrow l^+ \nu K^-)$ for large times t CPLEAR has found [6]

$$\text{Re } \cos \theta = (6.0 \pm 6.6 \pm 1.2) \times 10^{-4}. \tag{15}$$

From the decay rate evolution they have inferred

$$\begin{aligned}\text{Im } \cos \theta &= (-3.0 \pm 4.6 \pm 0.6) \times 10^{-2}, \\ \frac{1}{2} \text{Re } (x_l - \bar{x}_l) &= (0.2 \pm 1.3 \pm 0.3) \times 10^{-2}, \\ \frac{1}{2} \text{Im } (x_l + \bar{x}_l) &= (1.2 \pm 2.2 \pm 0.3) \times 10^{-2} .\end{aligned}\tag{16}$$

While there is no sign of **CPT** violation in any of these observables, the bounds of Eq.(16) are not overly restrictive.

Another input is provided by the charge asymmetry in semileptonic K_L decays for which the general expression reads as follows:

$$\begin{aligned}\delta_{\text{Lept}} &= \frac{\Gamma(K_L \rightarrow l^+ \nu \pi^-) - \Gamma(K_L \rightarrow l^- \nu \pi^+)}{\Gamma(K_L \rightarrow l^+ \nu \pi^-) + \Gamma(K_L \rightarrow l^- \nu \pi^+)} \\ &= \text{Im } \phi - \text{Re } \cos \theta - \text{Re } (x_l - \bar{x}_l) - 2\text{Re } y_l.\end{aligned}\tag{17}$$

CPT violation, if it exists, is most likely to surface in M_{12} , which is of second order in the weak interactions. It is then natural to assume *semileptonic* decay amplitudes to conserve **CPT**, which is fully consistent with Eq.(16), but not confirmed to the required level:

$$x_l - \bar{x}_l = 0, \quad \text{or} \quad y_l = 0.\tag{18}$$

With this *assumption*, and from the data [7]

$$\delta_{\text{Lept}} = (3.27 \pm 0.12) \times 10^{-3}.\tag{19}$$

one obtains

$$\text{Im } \phi - \text{Re } \cos \theta = (3.27 \pm 0.12) \times 10^{-3}\tag{20}$$

and infers from Eq.(15)

$$\text{Im } \phi = (3.9 \pm 0.7) \times 10^{-3},\tag{21}$$

showing that **T** is violated in kaon dynamics.

This result can be stated more concisely as follows [9]:

$$A_T \simeq A_{\text{CP}} = (6.6 \pm 1.3 \pm 1.0) \times 10^{-3}.\tag{22}$$

In order to get a result independent of the assumption that *direct semileptonic* kaon decays obey **CPT** symmetry, the CPLEAR collaboration has employed constraints from the Bell-Steinberger relation to deduce the bound [2]

$$\frac{1}{2} \text{Re } (x_l - \bar{x}_l) - \text{Re } y_l = (-0.4 \pm 0.6) \times 10^{-3},\tag{23}$$

which again is fully consistent with **CPT** invariance of the semileptonic decays. This results in establishing violation of **T** symmetry – provided the assumption mentioned above is valid.

4 Phases of η_{+-} & η_{00} and CPT

4.1 Basic Expressions

Manipulating Eq.(10) we obtain through $\mathcal{O}(\phi)$ and $\mathcal{O}(\cos\theta)$

$$|\eta_{+-}| \frac{\Delta\Phi}{\sin\phi_{SW}} = \left(\frac{M_{\bar{K}} - M_K}{2\Delta M} + R_{direct} \right) \quad (24)$$

$$\begin{aligned} \Delta\Phi &\equiv \frac{2}{3}\phi_{+-} + \frac{1}{3}\phi_{00} - \phi_{SW} \\ R_{direct} &= \frac{1}{2}\text{Re } r_A - \frac{ie^{-i\phi_{SW}}}{\sin\phi_{SW}} \sum_{f \neq [2\pi]_0} \epsilon(f) \\ r_A &\equiv \frac{\bar{A}_0}{A_0} - 1, \quad \phi_{SW} \equiv \tan^{-1} \frac{2\Delta M}{\Delta\Gamma} \\ \epsilon(f) &= e^{i\phi_{SW}} i \cos\phi_{SW} \frac{\text{Im } \Gamma_{12}(f)}{\Delta\Gamma}. \end{aligned} \quad (25)$$

Since **CPT** symmetry predicts $M_K = M_{\bar{K}}$ and $\text{Re } r_A = \mathcal{O}(\xi_0^2)$, where $\xi_0 = \arg A_0$, it implies $|\Delta\Phi| = 0$ to within the uncertainty given by $|\sum_{f \neq [2\pi]_0} \epsilon(f)|$; the latter sum thus represents the theoretical ‘noise’.

4.2 Estimating $\sum \epsilon(f)$

The major kaon decay modes fall into two classes, namely flavor-*nonspecific* or flavor-specific channels.

- With $A_f = \langle f | H_W | K^0 \rangle$ and $\bar{A}_f = \langle f | H_W | \bar{K}^0 \rangle$, we have, to first order in **CP** violation,

$$\text{Im } \Gamma_{12}(f) = i\eta_f \Gamma(K \rightarrow f) \left(1 - \eta_f \frac{\bar{A}_f}{A_f} \right), \quad (26)$$

for **CP** eigenstates with eigenvalue η_f . $\text{Im } \Gamma_{12}(f) \neq 0$ can hold only if $\bar{A}_f \neq \eta_f A_f$, i.e. if there is *direct CP* violation in the channel f .

Using data on ϵ' , $\text{Br}(K_{L,S} \rightarrow 3\pi)$ and

$$\text{Im } \eta_{+-0} = \left(-2 \pm 9 \begin{smallmatrix} +2 \\ -1 \end{smallmatrix} \right) \times 10^{-3}, \quad \text{Im } \eta_{000} = 0.07 \pm 0.16 \quad [10, 11], \quad (27)$$

where

$$\eta_{+-0,000} \equiv \frac{1}{2} \left(1 + \frac{q_1}{p_1} \frac{\bar{A}(\pi^+\pi^-\pi^0, 3\pi^0)}{A(\pi^+\pi^-\pi^0, 3\pi^0)} \right), \quad (28)$$

we obtain

$$\begin{aligned} |\epsilon(3\pi^0)| &< 1.1 \times 10^{-4} \\ |\epsilon([2\pi]_2)| &\simeq 0.28 \times 10^{-6} \\ |\epsilon((\pi^+\pi^-\pi^0)_{\mathbf{CP} - [+]})| &< 5 [0.2] \times 10^{-6}, \end{aligned} \quad (29)$$

- Allowing for a violation of the $\Delta Q = \Delta S$ rule in semileptonic decays as expressed by $x_l \equiv \frac{\langle l^+\nu\pi^-|\mathcal{H}_W|\overline{K}\rangle}{\langle l^+\nu\pi^-|\mathcal{H}_W|K\rangle}$, we find

$$|\epsilon(\pi l\nu)| \leq 4 \times 10^{-7}. \quad (30)$$

4.3 Quantifying CPT Tests

With the measured values for the phases ϕ_{+-} , $\phi_{00} - \phi_{+-}$, and ϕ_{SW} we arrive at a result quite consistent with zero [7, 12, 18]:

$$\Delta\Phi = 0.01^\circ \pm 0.7^\circ|_{exp.} \pm 1.5^\circ|_{theor.}, \quad (31)$$

i.e., the phases ϕ_{+-} and ϕ_{00} agree with their **CPT** prescribed values to within 2° . **CPT** invariance is thus probed to about the $\delta\phi/\phi_{SW} \sim 5\%$ level. The relationship between ϕ_{+-} , ϕ_{00} on one side and ϕ_{SW} on the other is a truly meaningful gauge; yet the numerical accuracy of that test is not overwhelming. The theoretical error can be reduced significantly by making quite reasonable assumptions on **CP** violation; however, we refrain from doing so based on our belief that assuming observable **CPT** breaking is not very reasonable to start with.

In Eq.(31), the *theoretical* uncertainty $\sum_f \epsilon(f)$ provides the limiting factor for this test[13]; it is dominated by $K \rightarrow 3\pi^0$. Future experiments could reduce the uncertainty by a factor of up to two [11].

Alternatively we can state

$$\frac{M_{\overline{K}} - M_K}{2\Delta M} + \frac{1}{2}\text{Re } r_A = (0.06 \pm 4.0|_{exp} \pm 9|_{theor}) \times 10^{-5}. \quad (32)$$

Yet ΔM does not provide a meaningful calibrator; for it arises basically unchanged even if **CP** were conserved while the latter would imply $M_{\overline{K}} - M_K = 0$ and $r_A = 0$ irrespective of **CPT** breaking.

The often quoted truly spectacular bound (for $R_{direct} = 0$)

$$\frac{M_{\overline{K}} - M_K}{M_K} = (0.08 \pm 5.3|_{exp}) \times 10^{-19} \quad (33)$$

definitely overstates the numerical degree to which **CPT** invariance has been probed. M_K is not generated by weak interactions and thus cannot serve as a meaningful yardstick.

In summary: while no hint has been found indicating a limitation to **CPT** symmetry, the *experimental* evidence for it is far from overwhelming:

- Comparing the phases of η_{+-} and η_{00} with the superweak phase constitutes a meaningful test of **CPT** symmetry. Yet there is a ‘noise’ level of about 2° that cannot be reduced significantly [11].
- Relating the bound on the difference $|M_{\bar{K}} - M_K|$ to the kaon mass itself is extremely impressive numerically – yet meaningless.
- When entertaining the idea of **CPT** violation, we should not limit our curiosity to a single quantity like $\Delta\Phi$ (or equivalently $M_{\bar{K}} - M_K$).
- Finally, the reader should be reminded that **CPT** symmetry implies $\Delta\Phi \ll \phi_{SW}$ but the converse does not follow.

5 Consequences in a **T** Conserving World

5.1 Reproducing η_{+-}

Assuming nature to conserve **T**, which implies $\phi = 0$, see Eq.(7), we have:

$$\begin{aligned} \frac{|\eta_{+-}|\Delta\Phi}{\sin\phi_{SW}} &= -\frac{M_{11} - M_{22}}{2\Delta M} + \frac{1}{2}r_A, \\ \frac{|\eta_{+-}|}{\cos\phi_{SW}} &= -\frac{\Gamma_{11} - \Gamma_{22}}{4\Delta M}\text{tg}\phi_{SW} - \frac{1}{2}r_A. \\ \text{Re } \cos\theta &= -\frac{M_{11} - M_{22}}{\Delta M}\sin^2\phi_{SW} + \frac{1}{2}\frac{\Gamma_{11} - \Gamma_{22}}{\Delta M}\sin\phi_{SW}\cos\phi_{SW}. \end{aligned} \quad (34)$$

Inserting the values of η_{+-} , ϕ_{SW} and Eq.(15) we can solve for the three unknowns:

$$\begin{aligned} \frac{M_{11} - M_{22}}{\Delta M} &\simeq r_A \simeq (-3.9 \pm 0.7) \times 10^{-3} \\ \frac{\Gamma_{11} - \Gamma_{22}}{\Delta M} &\simeq (-5.0 \mp 1.4) \times 10^{-3}. \end{aligned} \quad (35)$$

The solution is very *unnatural* – Eq.(35), for example, requires cancellation between **CPT** violating $\Delta S = 1$ and 2 amplitudes. Yet however unnatural they may be, we must entertain this possibility unless we can exclude it empirically.

As a side remark, we mention that if we invoke the Bell-Steinberger relation in its usual form – meaning that kaon decays are effectively saturated by the $K \rightarrow 2\pi, 3\pi, l\nu\pi$ channels, then we have an additional relation [16]:

$$\frac{1}{2}r_A \approx -\frac{\Gamma_{11} - \Gamma_{22}}{4\Delta M}; \quad (36)$$

i.e, Eq.(34) then implies $\eta_{+-} \simeq 0$. This is not surprising since these known modes do not exhibit any sign of **CPT** violation. But, as we have remarked before, in testing **CPT** we want to stay away from invoking saturation by the known channels.

5.2 $K \rightarrow \pi\pi$

Where should such a large **CPT** violation show its face? Imposing $r_A \neq 0$ raises the prospects of unacceptably large direct **CP** violation in $K_L \rightarrow \pi\pi$. Eq.(10) can be reexpressed as follows:

$$\epsilon \simeq \frac{1}{\sqrt{1 + \left(\frac{\Delta\Gamma}{2\Delta M}\right)^2}} e^{i\phi_{SW}} \left(-\frac{\text{Im}M_{12}}{\Delta M_K} + \xi_0 \right) \quad (37)$$

$$\epsilon' = \frac{1}{2\sqrt{2}} \omega e^{i(\delta_2 - \delta_0)} \frac{q_2}{p_2} \left(\frac{\bar{A}_0}{A_0} - \frac{\bar{A}_2}{A_2} \right) \quad (38)$$

If **T** is conserved, $\frac{q_2}{p_2} \left(\frac{\bar{A}_0}{A_0} - \frac{\bar{A}_2}{A_2} \right)$ is real and Eq.(38) then tells us [7]

$$\arg \left(\frac{\epsilon'}{\epsilon} \right) = \delta_2 - \delta_0 - \phi_{SW} \simeq -(85.5 \pm 4)^\circ. \quad (39)$$

Therefore

$$\begin{aligned} \text{Re} \frac{\epsilon'}{\epsilon} &\simeq \cos(\delta_2 - \delta_0 - \phi_{SW}) \cdot \frac{|\omega|}{2\sqrt{2}|\eta_{+-}|} \cdot |\Delta_0 - \Delta_2| \\ &= 0.035 \cdot \left(0.087^{+0.061}_{-0.078} \right) \cdot \left| \frac{r'_A}{r_A} - 1 \right| = \left(3.0^{+2.2}_{-2.7} \right) \cdot 10^{-3} \cdot \left| \frac{r'_A}{r_A} - 1 \right| \end{aligned} \quad (40)$$

where

$$r'_A \equiv \frac{\bar{A}_2}{A_2} - 1 \quad (41)$$

Some remarkable features can be read off from this expression:

- For

$$\delta_2 - \delta_0 - \phi_{SW} = 90^\circ \quad (42)$$

which is still allowed by the data, one obtains

$$\text{Re} \frac{\epsilon'}{\epsilon} = 0. \quad (43)$$

As far as $K \rightarrow \pi\pi$ is concerned this amounts to a superweak scenario!

- The empirical landscape of **CP** violation has changed *qualitatively*: KTeV, confirming earlier observations of NA 31, has conclusively established the existence of direct **CP** violation [8]:

$$\text{Re} \frac{\epsilon'}{\epsilon} = (2.80 \pm 0.30 \pm 0.28) \cdot 10^{-3} \quad (44)$$

Including previous data and preliminary results from NA 48 one arrives at a world average of

$$\text{Re}\frac{\epsilon'}{\epsilon} = (2.12 \pm 0.28) \cdot 10^{-3} \quad (45)$$

This can be reproduced with a ‘canonical’ $r'_A = 0$, but only for a very narrow slice in the phase of ϵ'/ϵ , namely

$$\delta_2 - \delta_0 - \phi_{SW} \simeq -(86.5 \pm 0.5)^\circ. \quad (46)$$

- The dominant uncertainty here enters through the phase shifts $\delta_{0,2}$. If $\delta_2 - \delta_0 - \phi_{SW}$ falls outside the range of Eq.(46), then $r'_A \neq 0$ is needed to reproduce $\text{Re}(\epsilon'/\epsilon)$. As an illustration consider $\delta_2 - \delta_0 - \phi_{SW} = 80^\circ$. In that case $1/2 \leq r'_A/r_A \leq 5/6$ had to hold to obtain $1 \cdot 10^{-3} \leq \text{Re}(\epsilon'/\epsilon) \leq 3 \cdot 10^{-3}$. Hence $r'_A \sim -(2 \div 4) \cdot 10^{-3}$. More generally if

$$\delta_2 - \delta_0 - \phi_{SW} \leq 83^\circ \quad (47)$$

then the observed value of $\text{Re}(\epsilon'/\epsilon)$ would imply

$$r'_A \leq -10^{-3} \quad (48)$$

if **T** is conserved.

- This would have a dramatic impact on $K^\pm \rightarrow \pi^\pm \pi^0$ decays. For Eq.(48) implies a sizeable **CPT** asymmetry there

$$\frac{\Gamma(K^+ \rightarrow \pi^+ \pi^0) - \Gamma(K^- \rightarrow \pi^- \pi^0)}{\Gamma(K^+ \rightarrow \pi^+ \pi^0) + \Gamma(K^- \rightarrow \pi^- \pi^0)} > 10^{-3} \quad (49)$$

With **CPT** symmetry we predict here a direct **CP** asymmetry of at most $\mathcal{O}(10^{-6})$ due to electromagnetic corrections. Thirty year old data yield $(0.8 \pm 1.2) \cdot 10^{-2}$. Upcoming experiments will produce a much better measurement.

5.3 $K_L \rightarrow \pi^+ \pi^- e^+ e^-$

If the photon polarization $\vec{\epsilon}_\gamma$ in $K_L \rightarrow \pi^+ \pi^- \gamma$ were measured, we could form the **CP** and **T** odd correlation $P_\perp^\gamma \equiv \langle \vec{\epsilon}_\gamma \cdot (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}) \rangle$. A more practical realization of this idea is to analyze $K_L \rightarrow \pi^+ \pi^- e^+ e^-$ which proceeds like $K_L \rightarrow \pi^+ \pi^- \gamma^* \rightarrow \pi^+ \pi^- e^+ e^-$. It allows to determine a **CP** and **T** odd moment $\langle A \rangle$ related to P_\perp^γ by measuring the correlation between the $\pi^+ \pi^-$ and $e^+ e^-$ planes. This effect was predicted to be [14]

$$\langle A \rangle = (14.3 \pm 1.3)\% \quad (50)$$

and observed by KTeV [15]:

$$\langle A \rangle = (13.6 \pm 2.5 \pm 1.2)\% \quad (51)$$

It is mainly due to the interference between the bremsstrahlung process $K_L \Rightarrow K_{\mathbf{CP}+} \rightarrow \pi^+\pi^- \rightarrow \pi^+\pi^-\gamma^*$ and a one-step M1 reaction $K_L \rightarrow \pi^+\pi^-\gamma^*$. The former is **CP** violating and described by η_{+-} *irrespective* of the theory underlying **CP** violation.

It is a remarkable measurement since it has revealed a huge **CP** asymmetry in a rare channel that had not been observed before. While **T** odd correlations have been seen before in production processes and in nonleptonic hyperon decays, those – due to their sheer magnitude – had to be blamed on final state interactions; such an explanation turned out to be consistent with what we know about those. The quantity $\langle A \rangle$ on the other hand is a **T** odd correlation *sui generis* since it has a chance to be generated by microscopic **T** violation.

Yet the most intriguing question is what does this measurement teach us about **T** violation without reference to **CPT** symmetry? The answer is: Nothing really! For we have just shown – by giving a concrete example – that if we are sufficiently determined we can dial **CPT** violation in such a way that both the modulus and phase of η_{+-} are reproduced even with **T** invariant dynamics, and it is η_{+-} that controls $\langle A \rangle$.

5.3.1 A Comment on the Intricacies of Final State Interactions

It is well-known that a non-vanishing **T**-odd correlation does not necessarily establish **T** violation since final state interactions can induce it even if **T** is conserved. Yet even so the reader might be surprised by our findings that a value of $\langle A \rangle$ as large as 10% does not establish **T** violation. For it would be tempting to argue that in the case at hand final state interactions could not induce an effect even within an order of magnitude of the observed size. The argument might proceed as follows: $\langle A \rangle$ reflects the correlation between the $\pi^+ - \pi^-$ and the $e^+ - e^-$ planes; their relative orientation can be affected by final state interactions – but only of the electromagnetic variety; then $\langle A \rangle \gg 1\%$ could not arise.

If nothing else, our brute force scenario shows that such an argument is fallacious. This can be seen also more directly. As stated above there are two different contributions to $K_L \rightarrow \pi^+\pi^-e^+e^-$, namely the M1 amplitude which is **CP** neutral, and the bremsstrahlung one due to the presence of **CP** violation. One should note that the presence of the this second amplitude requires neither **T** violation nor final state interactions!

Let us assume for the moment that $\arg \eta_{+-} = 0$ were to hold. Ignoring final state interactions both in the M1 and the bremsstrahlung amplitudes one obtains $\langle A \rangle = 0$, since the former is imaginary and the latter real now. When the final state interactions are switched back on, they affect the two amplitudes differently. Interference can take place, and one finds (with $\arg \eta_{+-} = 0$) $\langle A \rangle \sim 8\%$. How can the orientation of the $\pi^+ - \pi^-$ and the $e^+ - e^-$ planes get shifted so much by strong final state interactions? The fallacy of the intuitive argument sketched above derives from its purely classical nature. In quantum mechanics it is not surprising at all

that phase shifts between coherent amplitudes change angular correlations.

6 Summary

In this note we have listed the information we can infer on **T** and **CPT** invariance from the data on kaon decays. Our reasoning was guided by the conviction that once we contemplate **CPT** breaking the notion of a reasonable or natural assumption starts to resemble an oxymoron.

Our findings can be summarized as follows:

- The presence of **T** violation in $\Delta S \neq 0$ dynamics has been shown without invoking **CPT** symmetry through the Kabir test performed by CPLEAR. Yet their analysis had to assume semileptonic kaon decays to be **CPT** symmetric or it had to impose the Bell-Steinberger relation in its conventional form. We do not view either assumption as qualitatively more sacrosanct than **CPT** symmetry.
- $\phi_{+-,00}$ lie within 2° of what is expected from **CPT** symmetry.
- A meaningful yardstick for calibrating bounds on limitations to **CPT** symmetry is provided by **CP** asymmetries. **CPT** breaking forces could – empirically – still be as large as few percent of **CP** violating forces.
- It is grossly misleading to calibrate the bound on $M_{\bar{K}} - M_K$ inferred from ϕ_{+-} , ϕ_{00} and ϕ_{SW} to the kaon mass.
- The measured values of η_{+-} and η_{00} provide us with little information on the level of **T** versus **CPT** violation. More specifically η_{+-} – both its modulus as well as its phase – can be reproduced with **T** invariant dynamics (unless one imposes the Bell-Steinberger relation):
 - This is achieved by carefully adjusting **CPT** violation in $\Delta S = 1\&2$ transitions.
 - The observed level of direct **CP** violation – $\eta_{+-} \neq \eta_{00}$ – is *not* a natural consequence of such a scenario. However it could arise due to a fine-tuning of $\delta_2 - \delta_0 - \phi_{SW}$ – which had to be viewed as completely accidental – or to a compensation of direct **CPT** violation in $K_L \rightarrow [\pi\pi]_0$ and $K_L \rightarrow [\pi\pi]_2$.
 - In the latter subscenario one is stuck with a **CPT** asymmetry in $K^\pm \rightarrow \pi^\pm \pi^0$ that could be up to $\text{few} \times 10^{-3}$ without upsetting any known empirical bound.
 - The KTeV observation of a large **CP** and **T** odd correlation in $K_L \rightarrow \pi^+ \pi^- e^+ e^-$ in agreement with theoretical predictions is highly intriguing, yet does *not* constitute an unequivocal signal for **T** violation. This has also been noted before [17] using a different line of reasoning.

- We are fully aware that our construction is purely ad-hoc without any redeeming theoretical feature. Nevertheless we do not view it as l'art pour l'art (or more appropriately non-art pour non-art):
 - We have shown by constructing an explicit counter-example that the **T** odd correlation observed in $K_L \rightarrow \pi^+ \pi^- e^+ e^-$ does *not* establish **T** violation without invoking the **CPT** theorem.
 - As a by-product we have found that $K^\pm \rightarrow \pi^\pm \pi^0$ could exhibit a **CPT** asymmetry large enough to become observable soon.

Finally we would like to add the remark that even negative searches for **CPT** violation in kaon transitions will *not* free us from the obligation to probe for such effects in beauty meson decays at the B factories.

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